Part of the difficulty in learning science and math is learning how to study appropriately and to actually learn the material, as opposed to simply being able to memorize everything you see. It can be particularly challenging to recognize when you don’t fully understand a concept even though you may be able to do associated problems. In a typical course, professors try to help you learn the different ways to approach a concept by having you do lots of problems. It is then your responsibility to think about all of the problems and to recognize how they are similar, how they differ, and what types of variations you might see on an exam. In practice, this conceptualization can be extremely difficult to manage.

This handout gives you guidance for applying a particular problem-solving method that has been shown to be effective preparation for exams in problem-solving courses. While the method is broadly applicable to all problem-solving courses (e.g., math, chemistry, biology, physics, economics, statistics, etc.), it is often easier to understand and apply the method when learned in the context of the courses you are taking. Thus, this handout uses a math example to specifically target those students who are in introductory math courses at Duke.

You are also encouraged to work directly with an ASIP instructor to master these study strategies and other academic skills.

We will focus on a sample question as a way of gaining insight about the problem-manipulation method. The question used in this handout was contributed by faculty in the Duke Math Dept. Even if you don’t remember this material, try the example anyway.

Suppose \( g(t) = A e^{-rt} \), where \( A > 0 \) and \( r > 0 \), represents the quantity of a drug in the bloodstream at time \( t \). Suppose also that the half-life of the drug’s presence in the bloodstream is \( H \). Answer the following questions. Your answers may involve the constants \( A, r, \) or \( H \).

(Note: We can make this question a little harder by saying: Express your answers using the constants \( A \) or \( H \), but not \( r \).)

(a) What is \( g(t) \) when \( t = H \)?
(b) When is \( g(t) = \frac{A}{8} \)?

Whether you think you understand the question or not, it is important to learn how to read questions so that you don’t make silly errors. The first thing to do is to verify that you actually know what all of the terms mean. For example, what is a half-life?

Next, verify that you actually understand what the question is asking for. In this case, the question asks you to give your answer using the constants \( A, r, \) or \( H \)…why? Often, you can zero in on the meaning by asking yourself the question, “As opposed to what?”

Here is a potential logic sequence for part (a) written out for comparison to your own deductive reasoning (next page):
OK. I am being asked to determine the quantity of drug when the time ($t$) is equivalent to a single half-life ($H$). Well, a half-life is the time it takes for a drug to reach one-half of the original concentration ($A$). So, clearly, $g(t) = 1/2$ here, or more accurately, $g(t) = A/2$, since that is the time it takes for the drug to reach one-half of the original concentration.

The key here is to get away from the formula, which does not help you other than to clarify the nature of the relationships among the variables. You are told in the question that $H$ is equivalent to the value of the half-life. In other words, if you were to somehow calculate the actual half-life of this drug, that value is equivalent to $H$. Make sure to translate the component questions into words. Here, you can read the question as: When the drug has been in the bloodstream for exactly the length of time as its half-life, how much of the drug is still left? Obviously the answer is $1/2$! You can do slightly better than that here because the starting quantity is $A$, as specified by the formula, so $A/2$ is the complete answer.

Note that you could theoretically solve the equation, but since you are using variables instead of numbers, this is actually a rather messy process.

For part (b):

*That is 3 half-lives or $t = 3H$.*

Alternatively, you can solve for $A/8 = Ae^{-rt}$ for $t$. The logic is much the same as above. If you divide the value $A/2$ in half, twice, you get $A/8$, which means that you must have reached three half-lives.

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Many students are capable of doing this much, either with or without help. And most students would be happy to see that they got the answer correct and would move on to a new problem. However, you are missing a lot of opportunities to truly test your understanding of this problem when you just move on. Instead, there are a few follow-up exercises that you must do in order to be sure that you really get it.

**First: Ask yourself what concepts are being tested by the problem.**

There are a number of concepts that are relevant to this problem, and it is not necessary to think of every last possibility. These concepts include:

- Thinking in letters, as opposed to relying on numerical calculations.
- Half-lives.
- Common metrics (i.e., which variables are measurements of time).
- Etc.

**Next: Try to manipulate the problem in some way that challenges your understanding of the concepts you just listed.**

Here are some possibilities:

i. Try other equivalencies. When is $g(t) = A/16$? $A/12$?

   What is $g(t)$ when $t = 10H$? $20H$? $0.5H$?

ii. Try new initial relationships. What if $H$ is the 1/3 life? What if you didn’t know $H$ (i.e., what if you were not told about a variable that represents the half-life)?
iii. Try to use other variables. Does the value of $r$ impact your answers to the questions above? Why or why not? Try plugging in reasonable numbers. Try to create a real-world scenario…. for example, what if you are a doctor in an ambulance with someone suffering a seizure. You have two drug choices. One drug is more effective at calming the seizures but has a half-life of 3 minutes. The other drug is less effective with the seizures but has a half-life of 10 minutes. Both drugs cause damage to other areas of the body and should be used sparingly. What information would you, as the doctor, want to know in order to decide which drug to use? What values would lead to you to prefer one drug over the other?

Obviously, you can perform similar explorations of the problem for any concepts you might have listed. Perhaps the most important question of all to ask yourself is the following:

**Under what conditions would your original answer to this problem be incorrect?**

Note that this process does not require advanced knowledge of math. You should be able to think up most of these manipulations on your own, though it can be especially useful to study in this manner with a group.

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**Some additional comments:**

- Studying in this manner will take more time than simply blasting through the problems. However, if you are not currently understanding the material or doing well on the exam (or even if you are), simply doing more problems will probably not be sufficient to improve your performance. **Ultimately, you can get far better understanding by manipulating one problem exhaustively than you can by doing ten different problems that test similar concepts.** The key difference is the amount of thought you put into the problem, especially with reference to the concepts that the problem is supposed to be testing.

- **When you manipulate problems in this manner, you are mimicking the process by which the professor designs the exams.** Professors usually use pre-existing problems as templates and then manipulate them in some way to further test certain concepts. By anticipating the concepts being tested, and then doing the manipulations yourself, there’s a good chance you’ll predict possible test questions by accident.

- **You’ll find that you remember how to do these problems for a much longer time than otherwise.** Retaining the knowledge is especially important when you have many other classes competing for your time and for final exams, when much of the material was covered many months previously.

- **You’ll find that you can relate the material you are learning to real-world situations more easily than otherwise, which can make the course more meaningful to you.** The ultimate payoff comes many years later when you realize that you actually learned something in your intro courses rather than simply trying to get through them with a high grade. It may be hard to remember in the thick of it, but ostensibly the reason to go to college is to learn something.
Quick flow chart for manipulating problems

Read the problem

Does the problem make sense?

Yes

Answer the problem. Check your answer to be sure it’s correct.

No

Examine the problem closely. Check your understanding of the terms. Try to rephrase the question.

Step 1

What concepts are covered by the problem?
1. __________________________
2. __________________________
3. __________________________
Etc.

Step 2

Manipulate the problem to test your understanding of each of the concepts you listed. Generally this requires making the problem more complicated. Answer the new problem(s).

Repeat as needed.

Step 3

Ask yourself the following questions:

“How can I change the problem so that I get a different answer from before?”

“What information matters? What doesn’t?”

Move on to a new problem…

i) Change the way the question is worded.
ii) Change the information given.
iii) Change other aspects of the problem.

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